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THE CRITICAL PATH METHOD: ITS FUNDAMENTALS

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Submitted in partial fulfillment of  
the requirements for the degree of

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IN  
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and

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## ABSTRACT

The Critical Path Method is a modern technique for the making and updating of business decisions in connection with the analyzing, planning, scheduling, and monitoring of large and complex projects. The intent of this research paper is to provide a fundamental treatise, written in the language of the layman, for the use of a businessman or project manager seeking a management technique of potential use to him.



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## CHAPTER I

### INTRODUCTION

The Critical Path Method (CPM), one of a host of related network management techniques developed and popularized since 1960, is a modern method for the making and updating of business decisions in connection with the analyzing, planning, scheduling, and monitoring of large and complex projects. In the furtherance of the management by exception principle, the CPM breaks a large project down into an ordered sequence of individual tasks. This permits the singular consideration of the level of contribution of each task to the overall project completion, thus assisting in a determination of where in the project to apply limited available resources toward its timely and efficient completion.

The intent of this paper is to weave some of the many writings about CPM into a comprehensive treatise, written in the language of the layman, for the use of a businessman or project manager seeking a management technique of potential use to him. The majority of the writings to date on CPM appear to fall into two general classes - those extolling the successes and virtues of the technique with little basic factual development or no reference to its limitations and those so enmeshed in mathematical or descriptive details that they neglect most practical considerations. This paper hopefully will provide a middle ground between these two extremes; it will describe the CPM in the simplest possible language and develop a sample project for illustrative purposes; it will cover both the advantages and limitations to be found in the utilization of CPM in project management; it will briefly describe the mathematical/linear programming basis of CPM; and it will attempt



to depict the relationship of CPM to the other members of the network family of techniques.

The authors make no claim to any novel or original contribution to the network science; they propose only to clarify what has already been written about CPM to the ultimate benefit of the previously-uninformed businessman or project manager who may have a legitimate and acute need for an application of the technique.



## CHAPTER II

### HISTORY AND PHILOSOPHY OF CPM

#### HISTORY

While 1958 can be thought of as the year of initiation of the science of network analysis the actual beginnings can be traced to the times of Frederick W. Taylor and Herman Gantt. Their major contribution to scientific production scheduling methods and an analytical approach to management problems culminated in the well known bar or Gantt Milestone Chart which portrays the degree of completion of each individual task in a program. An inherent lack in this approach, however, is its inability to reflect interrelationships between the various tasks and constraints as applied to the overall project. By the mid-1950's it was recognized that a more revealing and detailed analysis was required to be effective in the rapidly expanding world of modern business management.

In 1958 the duPont Company, in an effort to reduce maintenance and construction costs, assigned a study group to investigate the problem. The approach settled on by this group was the development of an improved scheduling technique as a means of reducing unproductive downtime and thereby achieving the desired savings. The results and findings of this group were reported in March 1959.<sup>1</sup>

Further developments of the technique and the establishment of basic criteria for the use of the new system resulted from the continued study conducted by Morgan R. Walker and James E. Kelley, Jr., (In papers presented to the 12th International Systems Meeting in October 1959<sup>2</sup> and to the Eastern Joint Computer Conference in December<sup>3</sup>, the methodology of CPM was first published). Due to their early efforts and interest Walker and Kelley are





generally recognized as the originators of CPM.

It is interesting to note that during this same period a second research group was conducting similar studies. Under the auspices of the Navy Department's Special Programs Office, efforts to improve the planning and control system for the development of the Fleet Ballistic Missile Program resulted in the establishment of the Program Evaluation and Review Technique (PERT).<sup>4,5</sup> There is much similarity between the two methods, the major difference being that PERT utilizes probabilities and multiple times whereas CPM relies entirely upon a single "best" time estimate.

Subsequently, because the general ground rules and operating criteria as developed by Walker and Kelley make the technique of CPM applicable to such a broad range of industries and project oriented situations, numerous later network analysis applications were developed and reported. A discussion of some of these approaches will be included in Chapter V.

A recently developed approach by Levy, Thompson, and Weist<sup>6,7</sup> provides a simplified process for the utilization of the CPM analysis, especially when used in conjunction with computers. Differing slightly from the Kelley-Walker method, this approach appears to offer the greatest potential for future adaptations of the CPM technology.

#### PHILOSOPHY

The basic objective of CPM is the development of a graphical representation of the complete project plan showing the relationships and interdependencies of all activities associated with the project from start to finish. In CPM terminology (See Appendix A) such a graph is referred to as a network diagram. Once such a network has been developed it becomes a relatively easy



matter for the manager, rapidly and effectively, to maintain overall control of the project for which he is responsible.

The major advantages in the use of CPM are as follows:

- A disciplined and well-organized planning effort is required.
- The ability to manage by exception becomes an actuality.
- Permits the accurate forecasting of resource requirements.
- Provides a pictorial view of all interdependencies and interrelationships among activities in full detail.
- Will quickly alert the manager to any possible future problem areas both in location and effect on the overall project.
- Allows the use of the Monte Carlo system or other simulation methods for estimating and predicting the results of planned changes.

The net results of CPM, when used properly, will be a project reflecting sound planning and scheduling; improved cooperation and coordination between production centers will result in an improved contribution to performance; action to be taken to correct problems can be pinpointed and directed toward the proper areas rapidly and efficiently; the delineation of responsibilities and relationships between production centers will result in the availability of tighter controls; with the entire project developed and displayed on the network diagram, the scope of the project becomes understandable in its entirety, and oversights and duplications of effort are readily recognized. The result, therefore, will be a much more realistic and efficient operation.

There are, however, definite limitations to the use of CPM methods.

- There must be a clearly definable start and finish to the project. Continuing programs do not lend themselves to this type of analysis.
- There must be a feasible and achievable objective to be accomplished.



- Each task or activity included in the project must be definable.
- The duration or time required for each activity must be estimable, at least within broad limits.
- There must be interdependencies among the activities.
- There must be more than one path through the network from start to finish.

These constraints merely reflect the characteristic attributed to CPM that routine and recurring operations do not adapt themselves to CPM methods.

In the development and use of a CPM analysis, efforts logically and necessarily divide into three distinct areas. These might well be identified as the planning, scheduling, and control phases of the method. It should be noted that one of the primary benefits to this approach is the separation of the planning and scheduling functions. The influence of the two upon each other when accomplished concurrently, as in most older management techniques, frequently results in an inaccurate and less efficient operating plan.

Planning -- The initial undertaking in a CPM analysis is the complete identification of the project in terms of its component parts. This breakdown may be carried to any degree desired but should be such that complete control of the operation is insured. Care should be taken not to overcomplicate the procedure by the inclusion of excessive numbers of applicable activities. Once such identification is completed, the individual tasks are then included in the development of the network diagram.

Scheduling -- After the diagram has been constructed each task is then evaluated in terms of a "best" estimated time span or duration. These times are then included on the diagram and used during the control phase for the maintenance of progress checks. During this phase there is also developed available time-cost trade offs by activity, the equating of time periods in



terms of resource values, for possible future use. At this point the critical path is then determined. This computation results in the evaluation of the longest path or paths through the network establishing the estimated completion time.

Control -- During this period reports and records are maintained to reflect an updated status of the project. Decisions concerning the utilization of available resources, expenditure of extra efforts to obtain a reduction in activity times along the critical path, and the recomputation of new critical paths are made by the manager during this phase. Slippages and other problems are corrected, using the information generated by the established procedures, resulting in the maintenance of proposed time schedules or the forecasting of new times of completion. It should be remembered, however, that the main purpose of the CPM analysis is to indicate how a project will be accomplished and not how it might be.





## CHAPTER II FOOTNOTES

- <sup>1</sup>M. R. Walker, and J. S. Sayer, "Project Planning and Scheduling," Report 6959, E. I. du Pont de Nemours & Company, Inc., Wilmington, Delaware, March 1959.
- <sup>2</sup>Ideas for Management, Papers and Histories presented at the 12th International Systems Meeting, Systems and Procedures Association, Edwards Brothers, Inc., Ann Arbor, Michigan, 1960, pp 403-411.
- <sup>3</sup>James E. Kelley, Jr., and Morgan R. Walker, "Critical-Path Planning and Scheduling: An Introduction," Mauchly Associates, Inc., Ambler, Pa., 1959.
- <sup>4</sup>PERT (Program Evaluation Research Task), Phase I Summary Report, Special Projects Office, Bureau of Ordnance, Department of the Navy, Washington 25, D. C., July 1958.
- <sup>5</sup>PERT (Program Evaluation Research Task), Phase II Summary Report, Special Projects Office, Bureau of Ordnance, Department of the Navy, Washington 25, D. C., September 1958.
- <sup>6</sup>F. K. Levy, G. L. Thompson, and J. D. Wiest, "Critical Path Method - A New Tool for Management," O.N.R. Research Memorandum No. 97, Pittsburgh 13, Pa., May 25, 1962.
- <sup>7</sup>\_\_\_\_\_, \_\_\_\_\_, and \_\_\_\_\_, "Mathematical Basis of the Critical Path Method," O.N.R. Research Memorandum No. 86, Pittsburgh 13, Pa., May 30, 1962.



## CHAPTER III

### A SAMPLE DEVELOPMENT OF A PROJECT BY CPM

In order for a project to lend itself to the scheduling control of the Critical Path Method, it must have the following essential characteristics: (1) the project consists of a well-defined collection of tasks which, when completed, mark the end of the project; (2) the tasks may be started and stopped independently of each other, within a given sequence; and (3) the tasks are ordered, i.e., performed in technological sequence.<sup>1</sup> Basically then, the Critical Path Method breaks a large project down into an ordered sequence of individual tasks, i.e., each task is specifically related to the task or tasks that immediately precede it, the task(s) that immediately succeed it, and the task(s) that can be accomplished concurrently. The heart of the technique and the graphic convention normally used in portrayal of the CPM is a network diagram. The diagram is most valuable as a means of visually and conceptually relating the complex of tasks in a project to one another and to overall project requirements; however, it is noted that computer programs are available which permit the necessary calculations to be made without reference to any graph or diagram.<sup>2</sup>

Consider, for purposes of illustration of the method, a simple project<sup>3</sup> for the manufacture of a machine component which consists of two parts, P and Q. Each part must be turned on a lathe, and Q must also be polished. Two types of raw materials, A and B, are needed. Figure 1 is a listing of the tasks required to complete the project with a best estimate of the time required for each. Note the addition of two pseudo-tasks "start" and "finish". Each requires zero time to carry out; "start" is a predecessor to all other tasks; "finish" is a successor to all other tasks; and both are used for



bookkeeping purposes only.

Figure 2 is the corresponding network diagram for the project. The method used in depicting this diagram differs in some respects from the representation used by James E. Kelley, Jr., and Morgan R. Walker. In the Kelley-Walker format, tasks are shown as arrows, and the arrows are connected by means of small circles (or nodes) that indicate sequence relationships. Thus all immediate predecessors of a given job connect to a node at the tail of the task arrow, and all immediate successor tasks emanate from the node at the head of the task arrow. In essence, then, a node marks an event - the completion of all tasks leading into the node. Since these tasks are the immediate prerequisites for all task arrows leading out of the node, all must be completed before any succeeding tasks can begin. In order to accurately portray all predecessor relationships, "dummy" tasks must often be added to the network diagram in the Kelley-Walker form.<sup>4</sup> (See Appendix B for further details of the Kelley-Walker rules.) The method described below is a composite of what has been used by many later CPM authors; it avoids the necessity of "dummy" tasks; it is easier to program for a computer; and it is simpler to explain and apply.

In Figure 2, the number to the far left of each block (early start time) represents the earliest time the task can be started, and the number to the immediate right of each block (early finish time) the earliest time it can be completed. The number to the immediate left of each block (late start time) represents the latest time the task can be started without delaying the project, and the number to the far right of each block (late finish time) the latest time it can be completed without delaying the project. The slack time for each task is the difference between its late start time and its early



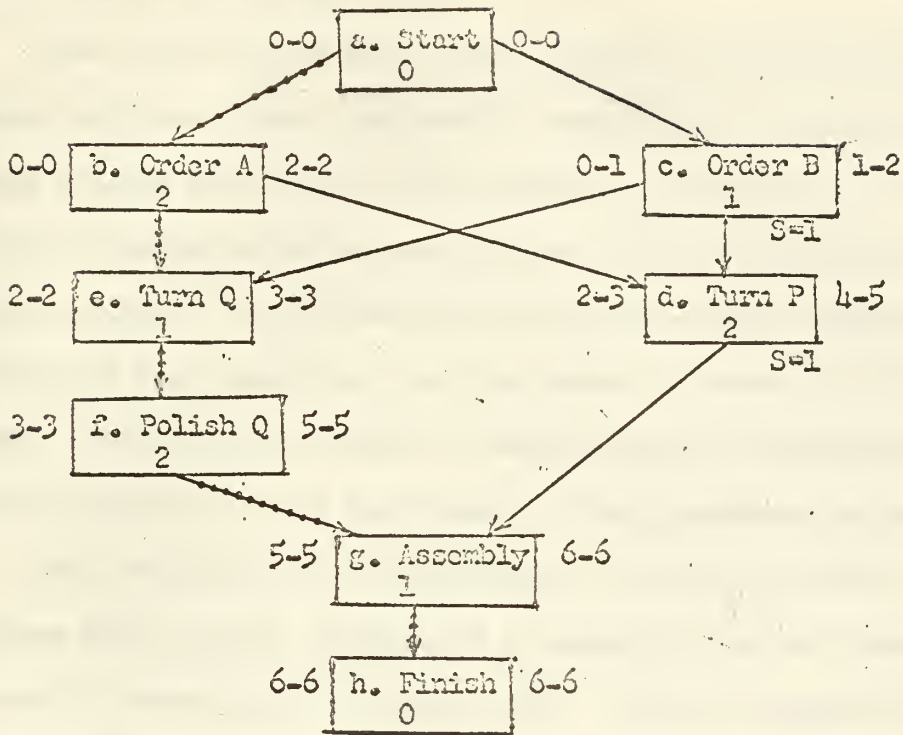
TASK	DESCRIPTION	IMMEDIATE PREDECESSORS	TIME REQUIRED
a	Start	None	0
b	Order A	a	2
c	Order B	a	1
d	Turn P	b,c	2
e	Turn Q	b,c	1
f	Polish Q	e	2
g	Assembly	d,f	1
h	Finish	g	0

FIGURE 1

TASK LISTING FOR SAMPLE PROJECT







The key to the notation used for each individual task block is shown below:

Early start time-Late s.t.	Task identification	Early finish time-Late f.t.
	Task time requirement	
	Slack time	

FIGURE 2  
NETWORK DIAGRAM FOR SAMPLE PROJECT



start time (or, equivalently, between its late finish time and its early finish time) and may be thought of as the available freedom or leeway in scheduling the particular task.

Thus, the critical path, which consists entirely of critical tasks (or tasks with zero slack), can now be identified as the longest continuum of tasks through the project (the dotted path, a-b-e-f-g-h, in Figure 2). Any delay in completion of any one of these tasks will delay completion of the entire project. At this point in the critical path analysis, all critical tasks have been identified, and the amount of slack in all the others is known. In addition, a graphic timetable has been constructed to facilitate further examination and improvement of task sequences and schedules.

Most projects can be accomplished in a number of ways ranging between minimum cost (usually accompanied by normal or optimal time) and minimum time (normally accompanied by maximum cost). Figure 3 graphically portrays the range of available alternatives in a typical project. Critical path scheduling permits an educated compromise between these extremes for the most efficient accomplishment of management goals. Figure 4 is a fictitious cost table illustrating the cost differences between the normal method of completing the project (as shown in Figures 1 and 2) and an alternative crash basis for the previously-developed example.

The original critical path from Figure 2, a-b-e-f-g-h, required 6 time units. The normal total project cost under these conditions would be \$100.00. From Figure 4, the cheapest way to shorten this path 1 time unit would be to accomplish Task b on a crash basis at an additional cost of \$5.00. Note, however, that, if this step is taken, path a-c-e-f-g-h also becomes critical, and any further calculations must now take account of two critical paths.



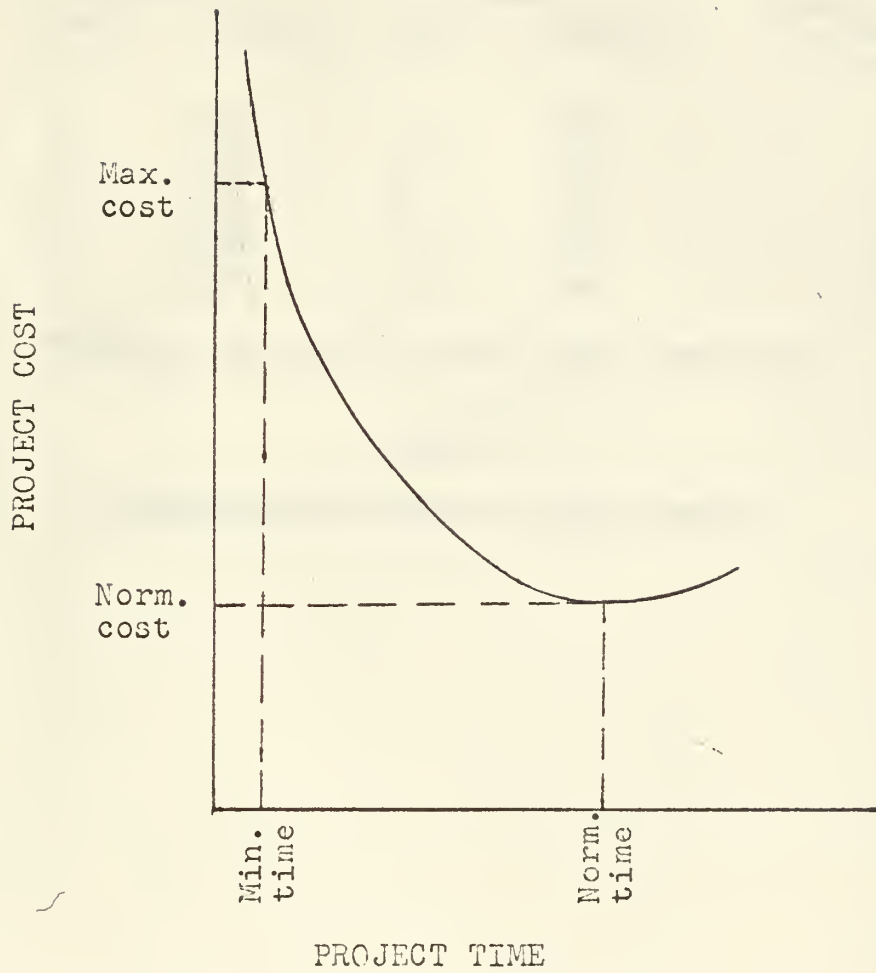


FIGURE 3

THE RANGE OF POSSIBLE COST  
ALTERNATIVES IN CRITICAL PATH SCHEDULING



Task	NORMAL		CRASH		Additional Costs Per Time Unit Saved
	Time	Dollars	Time	Dollars	
b*	2	\$10	1	\$15	\$5
c	1	\$10	1	\$10	--
d	2	\$20	1	\$30	\$10
e*	1	\$10	$\frac{1}{2}$	\$15	\$10
f*	2	\$30	1	\$60	\$30
g*	1	\$20	$\frac{1}{2}$	\$40	\$40

\*Denotes a critical task under normal conditions

FIGURE 4

COMPARATIVE COST TABLE FOR SAMPLE PROJECT





For example, both these paths could be shortened an additional  $\frac{1}{2}$  time unit by accomplishing Task e on a crash basis at an additional cost of \$5.00. Continuing these calculations, it can be readily seen that the absolute minimum time in which the project could be completed would be 3 time units at a total project cost of \$165.00. The network diagram would now appear as in Figure 5. It is noted that all four paths (a-b-e-f-g-h, a-c-e-f-g-h, a-b-d-g-h, and a-c-d-g-h) are now critical. Task d could still be shortened an additional  $\frac{1}{2}$  time unit at a cost of \$5.00, but to no theoretical avail in the scheduling phase, as the overall project time would still require a total of 3 time units. However, later on during the controlling phase of the project, it might still prove desirable to take this step if unforeseen slippages in earlier tasks have occurred.

The preceding example is illustrative of the time savings that can be realized from reducing the times of selected critical tasks. Of course, overhead costs have been neglected in this analysis. These fixed costs per project usually decrease as project time is shortened, and any actual application of the CPM would have to take these reductions into account, as well as such economic factors as the premium prices customers are willing to pay for the early delivery of end items. Actual results in the employment of CPM in industry indicate that significant reductions in project time can usually be realized for less than a 5% increase in total costs.<sup>5</sup>



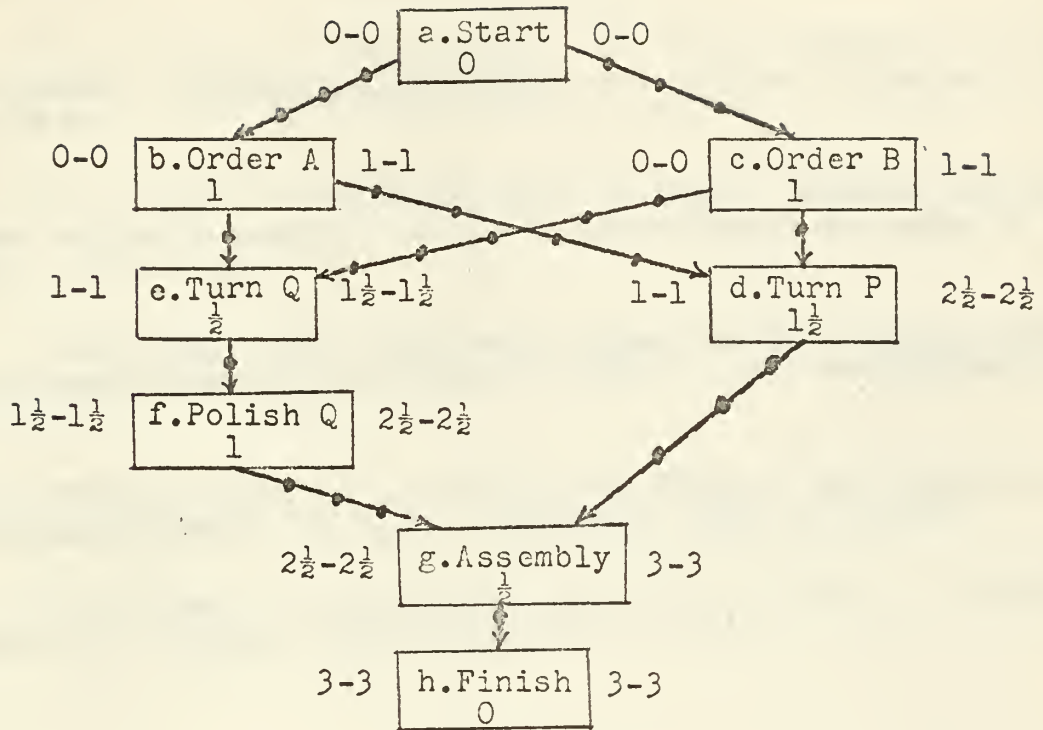


FIGURE 5

THEORETICAL MINIMUM-TIME NETWORK  
DIAGRAM FOR SAMPLE PROJECT



### CHAPTER III FOOTNOTES

<sup>1</sup>F. K. Levy, G. L. Thompson, and J. D. Wiest, "The ABCs of the Critical Path Method," Harvard Business Review, Vol. 41, No. 5 (Sept/Oct., 1963), pp. 98-99.

<sup>2</sup>F. K. Levy, G. L. Thompson, and J. D. Wiest, "Critical Path Method - A New Tool for Management," Office of Naval Research Memorandum No. 97, May 25, 1962, p. 2.

<sup>3</sup>Basic example taken from John G. Kemeny and others, Finite Mathematics with Business Applications (Englewood Cliffs, N. J.: Prentice-Hall, Inc., pp. 73-77.

<sup>4</sup>James E. Kelley, Jr., "Critical Path Planning and Scheduling," Operations Research, Vol 9, No. 3 (May-June, 1961), pp. 301-302.

<sup>5</sup>J. S. Sayer, J. E. Kelley, Jr., and Morgan R. Walker, "Critical Path Scheduling," Factory, July, 1960, p. 77.



## CHAPTER IV

### THE MATHEMATICAL BASIS OF CPM AND MANUAL VERSUS COMPUTER METHODS OF SOLUTION

The preceding example, considering a simple project of six tasks, is illustrative of the simplicity and straight forwardness of the CPM, but what about a complicated project consisting of 1600 rather than 6 tasks? Obviously, management must turn to a computer for assistance in such cases if it can. By the construction of mathematical models of the situation, electronic computers can be programmed to provide the same information for complicated projects that a network diagram provides for simpler ones.

The mathematical model upon which the CPM is based is a parametric linear program, incorporating sequence information, durations, and costs for each component of the project, and solvable via a primal-dual algorithm.<sup>1</sup> Like most other CPM theory, the mathematical model was largely developed by J. E. Kelley, Jr., utilizing both his own discovery of the relationship between parametric programming and the primaldual algorithm<sup>2</sup> and a variation of the Ford-Fulkerson flow algorithm.<sup>3</sup> Applying their own works on programming to Kelley's model, Charnes and Cooper have since developed quite a simple means of setting up the solution to a critical path problem in a standard primal-dual linear programming format.<sup>4</sup> To illustrate the simplicity of the Charnes-Cooper method, a network diagram in nodal form (Figure 6), a simplex format (Figure 7), the equivalent primal-dual sets of equations (Figure 8), and the computer solution (Figure 9) for the previously-developed example of Chapter III are illustrated on following pages.

Once the critical path problem is arranged in the standard linear programming form, there are many computer programs readily available which may be





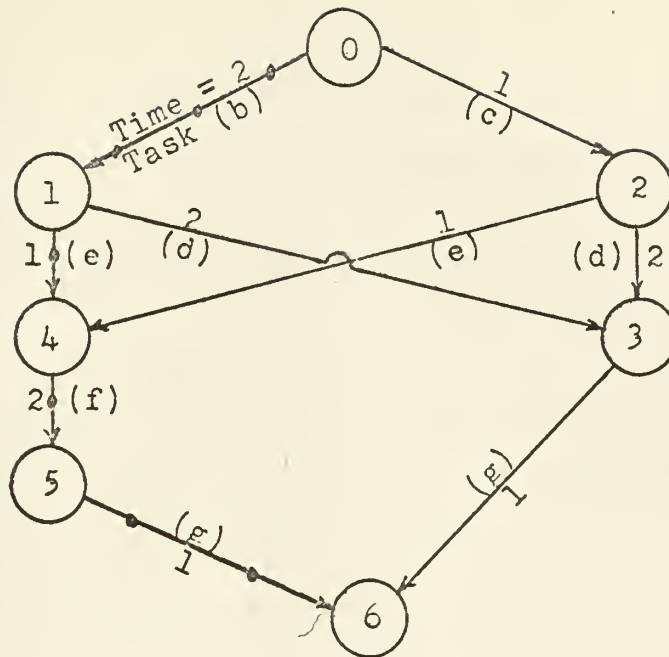


FIGURE 6

NETWORK DIAGRAM IN NODAL FORM

$t_{ij}$	2	1	2	1	1	2	1	2	1	Stipulations
links $w_i$	0-1 b	1-4 e	4-5 f	5-6 g	0-2 c	2-3 d	3-6 g	1-3 d	2-4 e	
0	1				1					1
1	-1	1						1		0
2					-1	1			1	0
3						-1	1	-1		0
4		-1	1						-1	0
5			-1	1						0
6				-1			-1			-1

FIGURE 7

CHARNES-COOPER SIMPLEX FORMAT



PRIMAL EQUATIONS

Maximize:  $2x_{01} + 1x_{14} + 2x_{45} + 1x_{56} + 1x_{02} + 2x_{23} + 1x_{36} + 2x_{13} + 1x_{24}$

subject to:

$$\begin{array}{rcl}
 x_{01} & + x_{02} & = 1 \\
 -x_{01} + x_{14} & & + x_{13} = 0 \\
 & -x_{02} + x_{23} & + x_{24} = 0 \\
 & & - x_{23} + x_{36} - x_{13} = 0 \\
 & & & - x_{24} = 0 \\
 -x_{14} + x_{45} & & & = 0 \\
 & - x_{45} + x_{56} & & = 0 \\
 & & - x_{56} & - x_{36} = -1
 \end{array}$$

and  $x_{ij} \geq 0$

DUAL EQUATIONS

Minimize:  $w_0$

$-w_6$

subject to:

$$\begin{array}{rcl}
 w_0 - w_1 & & \geq 2 \\
 & w_1 & - w_4 \geq 1 \\
 & & w_4 - w_5 \geq 2 \\
 & & & w_5 - w_6 \geq 1 \\
 w_0 & - w_2 & \geq 1 \\
 & w_2 - w_3 & \geq 2 \\
 & & w_3 - w_6 \geq 1 \\
 & w_1 - w_3 & \geq 2 \\
 & w_2 & - w_4 \geq 1
 \end{array}$$

in which the variables are not constrained in sign.

FIGURE 8

CHARNES-COOPER PRIMAL-DUAL EQUATIONS



The computer solution to the above primal problem would

appear as:  $2x_{01} + x_{14} + 2x_{45} + x_{56} = 6$

and to the dual as:  $w_0 - w_6 = 6.$

Since  $x_{01}$  in this notation is equivalent to task b in the notation of Chapter III,  $x_{14}$  to task e,  $x_{45}$  to task f, and  $x_{56}$  to task g, the solutions indicate that tasks (a)-b-e-f-g-(h) form the critical path of the network, each of them consuming the time of their respective coefficients, for a total elapsed project time ( $w_0 - w_6$ ) of 6 time units.

FIGURE 9

COMPUTER SOLUTION TO PRIMAL-DUAL EQUATIONS

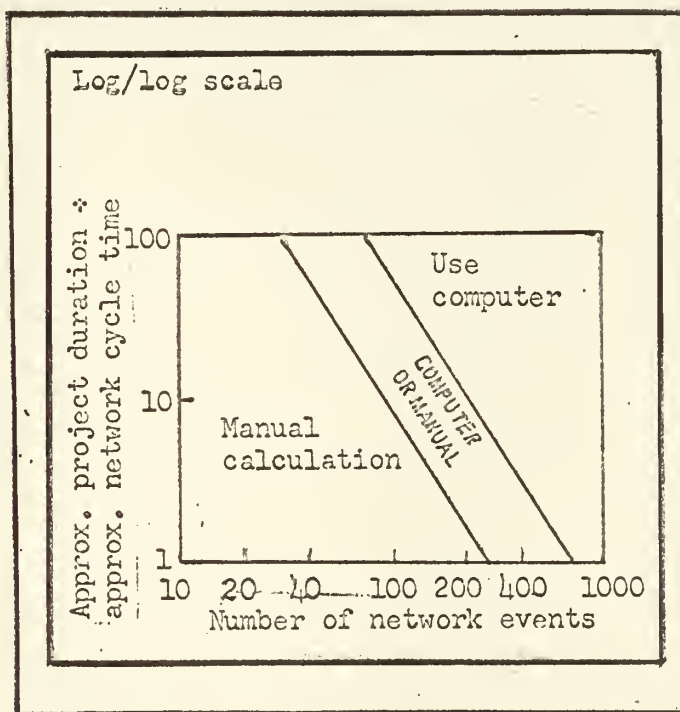


FIGURE 10

"FACTORY" GUIDE FOR THE SELECTION OF COMPUTER OR MANUAL METHODS OF PROJECT SCHEDULING BY CPM



utilized for its solution. (A list of selected critical path computer programs for both CPM and PERT is included as Appendix C.) There are numerous consulting and service firms who will provide these computations on a fee basis, or a business with a computer can have its own operating personnel trained in the fundamentals of critical path programming in about a week.<sup>5</sup> Figure 10, based on actual industry experience<sup>6</sup>, provides a rough guide to the relative feasibility of using computer or manual network methods in project scheduling by the CPM.





#### CHAPTER IV FOOTNOTES

<sup>1</sup>James E. Kelley, Jr., "Critical Path Planning and Scheduling: Mathematical Basis," Operations Research, Vol. 9, No. 3 (May-June, 1961), p. 296.

<sup>2</sup>J. E. Kelley, Jr., "Parametric Programming and the Primal-Dual Algorithm," Operations Research, Vol. 7 (1959), pp. 327-334.

<sup>3</sup>L. R. Ford and D. R. Fulkerson, "A Simple Algorithm for Finding Maximal Network Flows and an Application to the Hitchcock Problem," Canadian Journal of Mathematics, Vol. 9, (1957), pp. 210-218.

<sup>4</sup>A. Charnes and W. W. Cooper, "A Network Interpretation and a Directed Subdual Algorithm for Critical Path Scheduling," The Journal of Industrial Engineering, Vol. 13, No. 4, (July-August, 1962), pp. 213-219.

<sup>5</sup>J. S. Sayer, J. E. Kelley, Jr., and Morgan R. Walker, "Critical Path Scheduling," Factory, July, 1960, p. 77.

<sup>6</sup>Marvin Flaks and others, "Network Management Techniques," Factory, March, 1964, p. 91.



## CHAPTER V

### THE NETWORK FAMILY OF TECHNIQUES

The conduct of business in today's space-age atmosphere requires an ever more responsive and sophisticated capability in all fields of management. In the combination of the new concepts of time-cost relationships, critical path analysis, and improved reporting systems with the earlier Gantt Chart, milestone reporting systems, and network analysis, a versatile aid in the format of critical path analysis has been provided. The continuously expanding family of techniques, developed since 1958 and utilizing basic CPM concepts, is an indication of the potential benefits resulting from the intelligent use of such methods.

While the greatest emphasis in the development of these techniques has been provided by the Department of Defense, commercial applications are becoming more common as additional information and published accounts of successful uses become available. Far-sighted companies, led mainly by the major defense contractors, are recognizing the values inherent in such an approach. In the face of decreasing profit margins, scarce resources, and ever mounting development and production costs, the savings in valuable time to be realized assumes an increasing importance. As a result of these adaptations to meet individual and specific problems, there has been a steady addition to and broadening of network analysis.

In this chapter will be presented a representative sample of such applications; first as developed in and for military usage and second as a commercial by-product for other uses. While such a discussion is not intended to be exhaustive it will provide some indication of the wide range of application of network analysis.



## MILITARY

Faced with ever increasing defense costs and shrinking operating budgets, the Department of Defense has provided much of the necessary incentive towards the development of improved techniques in all areas of management. In view of the present policy of evaluating economic alternatives, to provide the maximum defense for a given dollar cost, the need for specific and accurate information is evident. The importance of such information is reflected in the requirement for the submission of critical path analysis data with submitted bids on defense contracts. Such information provides the best possible answers to many pertinent questions concerning and affecting the decision making process in the selection of the best alternative.

Scarce Resources. The attainment of both feasible and achievable objectives is, in large measure, dependent upon the proper allocation of available resources. This includes technical abilities, materials, and funds.

Leadtimes. While a program selected may be feasible and achievable, the time necessary for the development and production of the end product may be prohibitive.

Predicting and Controlling Costs. In the past, cost overruns to defense procurements have added greatly to defense spending, especially in the area of research and development. Because of the huge sums involved there is an obvious need for the most accurate forecasting of and the closest adherence to projected costs.

Design Changes and Modifications. Another practice which has added tremendously to defense costs in the past has been the extensive incorporation of alterations and modifications to originally contracted equipments. While such changes are generated rapidly under the present technological ability of industry, all too frequently such changes, included at additional



costs and sometimes resulting in excessive delays, have proven to be neither necessary nor warranted.

A valuable tool has become available for the use of planners and project managers with the development of the various critical path analysis techniques. With these and other tools the projecting and forecasting of future requirements and costs on a more realistic and factual basis has resulted in a much improved decision making process. An indication of the extensive use of these methods on the part of the military and other government agencies is given by Figure 11.

The following is a discussion of a selected sample of the various systems of analysis presently being used. These will give some indication of the evolutionary nature and constantly expanding and improving science of network analysis.

#### PERT (Program Evaluation Review Technique)

Developed during the same period as CPM this system is concerned mainly with improving the planning associated with research and development. The nomenclature and methods of PERT are very similar to those used in CPM: the selection of appropriate activities and events, the development of the network diagram, identification of the critical path, and improved progress reporting and control features. PERT, however, unlike CPM, utilizes probability theory in the scheduling process and develops three applicable times for each activity: a pessimistic, an optimistic, and a most likely time for completion. This is a major difference to the single most likely or best time estimate of CPM.

In addition, the original concept of PERT does not include a capability for the evaluation of activities in terms of cost as does CPM. As a result, expediting or speed-ups of projects under this method cannot be priced out in time-cost trade off fashion.





Some Governmental Agencies That Have Included The Use of CPM or PERT in Contract Specifications.<sup>1</sup>

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Department of the Navy:	Method Required:
Bureau of Naval Weapons	PERT
Special Projects Office	PERT, cost control
Bureau of Yards and Docks	CPM, cost control
Bureau of Ships	CPM and PERT
Department of the Air Force:	
Air Force Systems Command	PERT, cost control
Ballistic Systems Division	PERT, cost control
Electronic Systems Division	PERT
Department of the Army:	
Corps of Engineers	CPM, cost control
National Aeronautics and Space Admin: <sup>2</sup>	CPM, PERT, cost control
Federal Aviation Agency	CPM
General Services Administration	CPM

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The above list is only representative and not intended to be exhaustive.

FIGURE 11



## PERT/COST

A recent development has been the addition of a cost analysis capability in conjunction with PERT. This newer approach is referred to as PERT/COST.

In addition to the normal PERT information this method provides:

1. Individual activity cost estimates.
2. Contract values and realistic budget information.
3. A more efficient scheduling of available resources.
4. A capability to compare actual and budgeted costs and progress

at any time.

## LOB (Line of Balance)

While not a planning and scheduling procedure in the same sense as CPM and PERT, this method utilizes many identical concepts in the control of actual production scheduling. The major benefit to be gained is the graphical display of all production processes included in the project in the form of an updated status board. In this manner the completion status of the overall project is quickly determined and slippages and problem areas become readily apparent.

The primary advantage of the system is its ease of operation. Complex computers and mathematical formulas are not required. As a control record it can be maintained with a minimum of effort and is easily understood by all concerned.

The four major components of the system are: (1) a graph of the production and shipping schedules, (2) a time phased plan for production, (3) a bar chart indicating the status of performance within each control area, and (4) an understandable legend of the chart. When combined these present a complete picture of the production schedule and its latest status.



The procedure requires the definition of all component activities included in the production of an end item, starting with necessary procurement actions and ending in the shipment of the completed unit. Such steps as fabrication, testing, assembly, final testing, packing, and shipping are examples of the possible functions to be included in this technique. With the inclusion of duration time estimates for each step, it becomes a relatively simple matter to lay out the bar chart in such a manner that completion times and dates are readily indicated. The determination of event sequences provides the overall time requirements for the project. By maintaining up-dated progress and completion information for each activity, the manager is able to determine the current status of the project. Problem areas can be spotted early and corrective action taken before the entire project is affected and delayed.

Additional systems utilizing critical path analysis are:

**ABLE (Activity Balance Line Evaluation)**

A program status measuring, forecasting, and reporting system.

**COMET (Computer Operated Management Evaluation Technique)**

Designed for use in planning, scheduling, and monitoring the acquisition of electronic equipments and systems.

**CPA (Cost Planning and Appraisal)**

A system to assist equipment managers in developing a more effective means of managing cost-type contracts by integrating data on cost, time, and technology.

**IMPACT (Integrated Managerial Programming Analysis Control Technique)**

Provides progress reporting and program control for Government furnished material through use of a standardized PERT network and an equipment procurement factor data bank. The system includes milestone reporting, procurement



document control, item identification, efficiency forecasting, fund commitment, and obligation forecasting.

#### RAMPS (Resources Allocation and Multi-Project Scheduling)

An automated management technique for making the most of men, materials, and money. Based on CPM and PERT for control procedures.

For a more complete compilation of such systems see the work of Robert W. Miller, Director of Management Sciences, Raytheon Company.<sup>2</sup> In addition, for a well written description of these and other techniques, see the work of Vincent F. Callahan, Jr. and Lois C. Philmus.<sup>3</sup>

#### COMMERCIAL

In the commercial adaptations of business techniques where the efficient utilization of resources is reflected in the attainment of adequate profits, the search for operational savings is of the utmost necessity. The highly competitive nature of the free enterprise system metes out drastic penalties to the firm which does not fully control the time, cost, and performance factors of its production. The use of CPM methods has demonstrated the ability to effect such savings when properly conceived and utilized.

Having recognized the advantages of such analytical methods, numerous derivations and adaptations have been developed and perfected by various concerns. While the actual concepts have remained the same, such adaptations have concentrated upon the improvement in communications and manipulative systems associated with the rapid development and provision of necessary information.

#### LESS (Least Cost Estimating and Scheduling)<sup>4</sup>

Developed by the International Business Machine Corp., this computer oriented system has been applied to and is available for many varying industries and problem areas. Almost identical to the original CPM, this method





determines the fastest and most economical means of completing a project using arrow diagramming (network diagramming).

#### LOCS (Libroscope Operations Control System)

A computer based data accumulation and reporting system utilized in the design and production of electronic equipments. Control of purchasing and manufacturing cycles is maintained in the following manner:

1. Punched tapes are prepared whenever a document is initiated and automatically entered into a central computer.
2. Data collectors at key points within the company correlate such information and provide pertinent information to control points in the form of computer print outs. Such information as the start and finish of various processes, ordering or receipt of material, and shipping information is provided for the updating of appropriate analysis records.

Used in conjunction with CPM, PERT, or an appropriate variation, the system provides instantaneous updating of status information. This will be one of the first such systems to obtain the actual real-time savings necessary to efficient computer operation.

#### TRACE (Task Reporting and current Evaluation System)

Developed by Ling-Temco-Vought, Inc., this system is utilized for both military and other uses. The system is compatible with CPM techniques and provides information in such areas as costs, manpower, and forecasting. Used mainly as a reporting system, TRACE provides for the rapid maintenance of control records affecting widely separated and complex programs. The system utilizes leased lines, transceivers, and centralized computers.

This short summary of the systems in use today is indicative of the increasing acceptance of the CPM technology by industry. While minimal in numbers, this resume gives positive evidence of the application of these methods to a



wide variety of management and control areas. Such acceptance has proven the validity of the concepts when used under appropriate circumstances. Accounts of successful applications and generated savings are being reported steadily in various journals and publications. To the business firm where "time is money" such savings cannot be ignored.

Factory<sup>5</sup> provides the basic circumstances of two recent applications of CPM as follows:

Ford Motor Company (Detroit) applied critical path scheduling to a model changeover at a transmission plant. It cut project time 20% and saved "substantial" money. It was a big job, involving not only layout, machine installation, and facility tryout, but purchase of 200 pieces of machinery and tooling, 1500 gauges, and 60 types of raw material and finished parts. Management ran the conventional reporting system parallel with critical path scheduling, until everyone gained confidence in it.

The East Alton (Ill.) brass operations of Olin Metals has three massive projects under way at the same time. The three, which involve two outside engineering firms as well as Olin employees, include a new primary brass mill, expansion and modernization of two finishing mills about 500 miles apart, and the consolidation of scattered fabricating departments. All three projects maintain their own critical path schedules and feed information into a central control room at East Alton. Olin has been able to roll with several hard punches in the course of the four year undertaking, and credits critical path scheduling with enabling it to respond swiftly, on target, and at minimum cost whenever problems show up.

Harvard Business Review<sup>6</sup> provides another:

Du Pont, a pioneer in the application of the CPM to construction and maintenance projects, was concerned with the amount of downtime for maintenance at its Louisville works, which produces an intermediate product in the neoprene process. Analyzing the maintenance schedule by the CPM, Du Pont engineers were able to cut downtime for maintenance from 125 to 93 hours. The CPM pointed to further refinements that were expected to reduce total time to 78 hours. As a result, performance of the plant improved by about one million pounds in 1959, and the intermediate was no longer a bottleneck in the neoprene process.

Factory<sup>7</sup> provides, further, cases of successful applications of methods similar to CPM:

A major castings company used PERT to plan and control the purchase



and installation of a sand slinger in one of its foundries. The sand slinger is a major piece of equipment, involving considerable site preparation, engineering, and associated equipment and facilities. By adroit use of resources - especially manpower, which was color-coded by skill right on the network - the job was cut from 15 months to eight.

PERT smoothed out the move of an entire corporate division from White Plains, N. Y., to Oklahoma City, Okla., for Ling-Temco-Vought (Dallas) - about 1500 miles. A new hi-fi speaker plant had to be designed in 60 days and built in four months flat. Employees who declined the move had to be replaced by local people, hired and trained concurrently. Snags? Sure. But NMT helped the plant open for business on schedule.

Designing and building prototype equipment - especially on a tight schedule - is normally a good way to get ulcers. But the Martin Company (Orlando, Fla.) found that PERT works wonders on such projects. The company designed and built for its own use a unique machine that deposits "thin film" segments. It's used for producing complex electronic circuitry.

The network helped set time limits for design changes and alterations. It served as a task checklist for the procurement and prototype manufacturing groups. And it kept managers informed of day-to-day progress so they could take immediate, focused action. Among the pay-offs: two months were chopped off the schedule, and over 10 man-months were saved.

While the savings of these accounts are not assured in all instances, they are illustrative of the possibilities to be investigated. In the continuing progress and refinement of management practices, CPM methods and techniques certainly have a place.



## CHAPTER V FOOTNOTES

<sup>1</sup>J. J. Moder, and C. R. Phillips, Project Management with CPM and PERT, Reinhold Publishing Corp., New York, 1964, p. 11.

<sup>2</sup>R. W. Miller, Schedule, Cost, and Profit Control with PERT, McGraw-Hill Book Company, Inc., New York, 1963, pp 207-215.

<sup>3</sup>V. F. Callahan, Jr., and L. C. Philmus, Defense/Space Management Handbook, Callahan Publications, Washington, D. C., 1963, pp 14-49.

<sup>4</sup>M. C. Frishberg, "LESS Tells You How Project is Doing," Hydrocarbon Processing & Petroleum Refiner, Vol. 41, No. 2, February 1962, pp. 130-138.

<sup>5</sup>Marvin Flaks and others, "Network Management Techniques," Factory, March 1964, p. 91.

<sup>6</sup>F. K. Levy, G. L. Thompson, and J. D. Wiest, "The ABCs of the Critical Path Method," Harvard Business Review, Vol. 41, No. 5, (Sept/Oct., 1963), p. 100.

<sup>7</sup>Flaks, loc. cit.





## CHAPTER VI

### CONCLUSION: ADVANTAGES AND LIMITATIONS OF CPM

The preceding discussion of the CPM should not be construed as a proposal for its application to all large projects as a magical cure-all. In order for a project to lend itself to the analysis and control of the CPM, it must have the essential limiting characteristics noted in both Chapters II and III. These requirements immediately eliminate all open-end projects and continuous-flow processes, e.g., oil refining, from any analysis by critical path techniques.

Widely diverse kinds of projects do, however, lend themselves to analysis by the CPM, as is suggested by the following typical projects to which it might be applied:

1. The construction of a building.
2. Planning and launching a new product.
3. Installing and "debugging" a computer system.
4. Research and engineering design projects.
5. The scheduling of ship construction and repair.
6. The manufacture and assembly of a product under job-lot operations.
7. Turnaround maintenance projects in which an entire facility is shut down, overhauled, and put back into production.

When the project at hand fits the ground rules for application of the CPM on either a manual or computer basis, the major advantages of its utilization appear to boil down to these:

1. It separates the planning and scheduling phases of project preparation.
2. It focuses management attention on a critical few of the total project tasks.
3. It facilitates self-correction in that as errors in task time estimates are discovered, revised project graphs can be readily constructed, manually or by computer.
4. It places a price tag on change, thus enabling the greatest time savings for the least expenditure of dollars.



5. It is a basically simple technique, easy to learn and to apply, and highly adaptable to a wide variety of projects.

This is not to say that CPM, or for that matter any other network technique, is handed to management on a silver platter without inherent difficulties and costs. Some of the important disadvantages of CPM are these:

1. It is not flexible in regard to its sequence of events, i.e., one network allows one and only one such sequence. In order to provide for alternative sequences of events, alternate networks must be prepared and analyzed.
2. The time-cost tradeoff advantage of CPM analysis may, in some cases, simply not exist. Time and cost can vary independently as well as dependently, e.g., a technical breakthrough could gain time at little or no cost, and the failure of an end item on the final test stand could wipe out time gains which had been realized at heavy cost.
3. Resources - money, people, skills - are not always flexible, and it may not be possible to shift them to a more critical event as might be indicated advisable from project network analysis.
4. Managers and operating personnel must be trained in critical path fundamentals and operating techniques; and training effort costs both time and money.
5. Planning with CPM and other network techniques is costly. Some managers estimate that network planning is about twice as expensive as conventional planning approaches. More specifically, Aerojet-General Corporation (Azusa, California) allows 0.5% of the total cost of any project for PERT expense, and Ford Motor Company estimates that the cost is about 1% of the total cost of the first project and 0.5% of later ones conducted by the same people.<sup>1</sup>

However, the problems of increased planning costs and the other possible disadvantages which may accompany the introduction of CPM to a project should not preclude its use after a careful management determination of its appropriateness. Who can estimate the offsetting values of beating a competitor to market with a new product, of getting a new machine or plant into operation months ahead of the original schedule, or of improved communication and faster reaction to production problems? In conclusion, it is sufficient to



state that the Critical Path Method is a modern technique of tremendous potential benefit to a businessman or project manager when intelligently applied under appropriate circumstances.



CHAPTER VI FOOTNOTE

<sup>1</sup>Marvin Flaks and others, "Network Management Techniques," Factory, March, 1964, p. 91.





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## APPENDIX A

### BASIC CRITICAL PATH DEFINITIONS

- Activity** - a project element involving the expenditure of time and other resources.
- Dummy activity** - an activity included solely for bookkeeping purposes and utilizing no time or resources.
- Arrow** - the representation of an activity on the network diagram.
- CPM** - Critical Path Method.
- Critical Path** - the sequence of activities and events, from start to finish, requiring the maximum time during project completion.
- Event** - the beginning or completion of an activity or a group of activities.
- Float** - the amount of time an activity can be delayed without delaying completion of the overall project.
- Network** - a representation of a project in diagrammatic form which displays all activities and events and their mutual relationships.
- Node** - a representation of an event on a network diagram.
- Path** - any series of interconnected activities and events between the start and finish of a project.
- PERT** - Program Evaluation and Review Technique.
- Resource** - any contributing factor necessary for the completion of an activity within a project.
- Slack** - the amount of time an activity can be delayed without delaying completion of the overall project.



## APPENDIX B

### PROCEDURAL RULES FOR NETWORK ANALYSIS

In the development of the CPM network diagram there are a number of basic rules which need be applied. The use of such rules is necessary to insure a consistency of representation and, in some instances, for mathematical computation. While it is not mandatory to comply completely with these rules in a manual manipulation of the network, most computer programs require strict adherence.

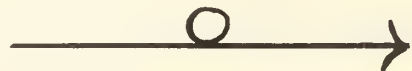
#### ACTIVITY

An activity is a required task to be performed in the completion of the project under control. Represented normally by an arrow, there must be both a start and finish to the activity. Other terms sometimes used to designate such activities are job and task. All refer to the expenditure of both time and resources.



#### DUMMY ACTIVITY

The so-called "dummy" activity is one wherein there is a requirement for neither resources nor time. The primary use of a "dummy" activity is to indicate a sequence or completion relationship between two activities when there is no specific dependency of one upon the other. This is especially useful when using a computer. Such activities are represented as dashed arrows or solid arrows with zero time indicated.





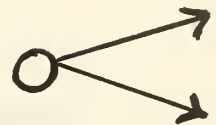


## EVENT

An event is the beginning or ending of any activity and represents a given point in time. Usually represented as a circle on the diagram the event is sometimes referred to as a "node" or "connector". When an event represents a multiple ending of two or more activities it is referred to as a "merge" event, and when it represents a multiple beginning for two or more activities as a "burst" event.



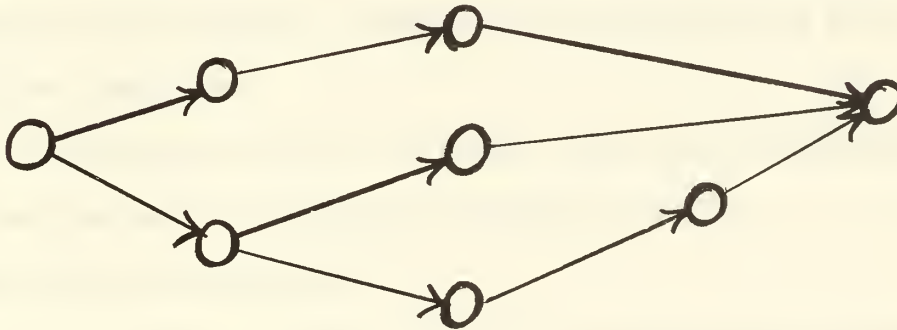
merge



burst

## NETWORK

The network is the graphical representation of all activities and events included in the overall project. A synonym sometimes used is the "arrow diagram".



## RULES:

1. No succeeding activity may begin prior to the completion of all activities which directly precede it.
2. Arrows reflect sequences and precedences only and do not normally represent time to scale. (In exceptional cases the diagram may be prepared on a time-scaled basis.)



Rules 3, 4, and 5 are required for computer use and are generally followed in all cases as a matter of form.

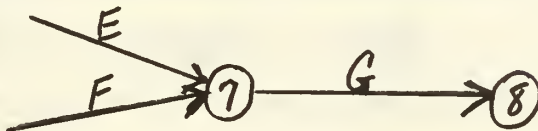
3. There must be no duplication of event numbers within a network.

4. Networks must begin with only one starting event (node) and end with one finish event (node).

5. There may be only one activity directly connecting any two events (nodes).

**EXPLANATION OF RULES.**

Rules 1. and 2. can be explained by the following diagram:



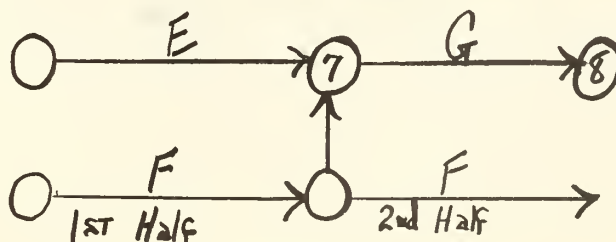
Before activity G can begin, activities E and F must be completed.

(Rule 1). There is no intent to indicate that activities E and F are to be completed simultaneously, since this is not included in Rule 1 and is not a necessary condition.

In accordance with Rule 2, events 7 and 8 are connected by activity G only and may not be connected by any other activity.

**COMMON ERRORS IN DIAGRAMMING:**

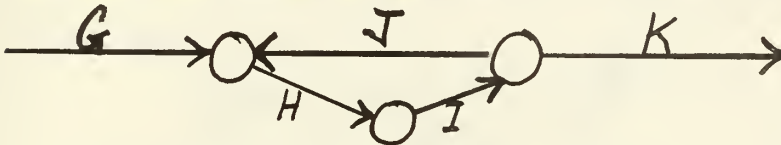
Rule 1 is the rule most frequently violated when constructing the network. In the example above, suppose activity G is dependent upon activity E completely but only upon the first half of activity F. The correct representation would then be as follows:





As can be seen an additional activity and event have been inserted to correct the mis-representation of complete dependency. This is referred to as a partial dependency.

A second common error encountered is the formation of a loop. As illustrated below, activities H, I, and J are presented such that I is dependent upon H, J is dependent upon I, and H is dependent upon J. Obviously, a loop has been formed and an impossible situation exists.

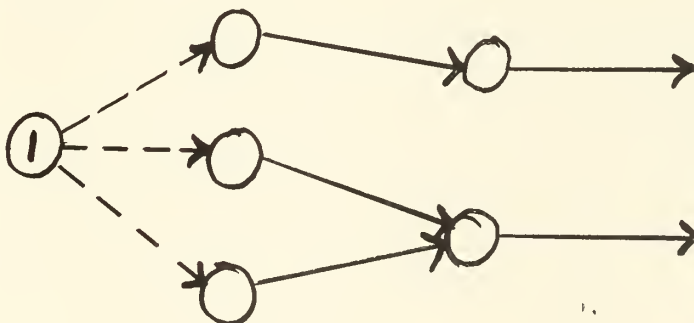


A violation of Rule 3 appears below:



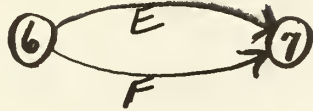
In using a computer the situation above could not be programmed. It is necessary, then, when employing computers, that the same event number not be used more than once. Some programs, in addition, require that each event be larger than any event preceding it.

Rule 4 requires that there be only one starting and ending event or node. Event 1 has been added below in order to join a number of concurrent first activities in order to conform with this requirement. (Note the use of dummy activities with no time or resource requirements to connect the affected events.)

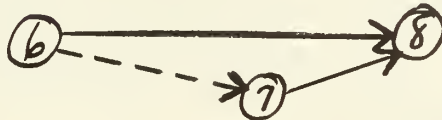




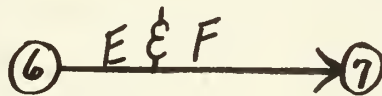
A violation of Rule 5 is illustrated below:



As can be seen there are two activities directly connecting the same events. To correct this situation a dummy activity and an additional event may be used.



A second, but less common, method would be to join the two activities into one. This method is seldom utilized due to the loss of desired detail within the diagram.







## APPENDIX C

### SELECTED CRITICAL PATH COMPUTER PROGRAMS<sup>1</sup>

No.	Computer Equipment	Contact for Further Information on Program	Capacity	Category and Comments
1	Burroughs 200	Supervisor of Programming Burroughs Computation Center Box 843 Paoli, Pennsylvania	500 activities 300 events	basic PERT
2	Burroughs 220	same as above	400 activities or more, depending on equip- ment	basic CPM for network system employing activities on nodes; utilizes calendar dates and permits random numbering of activities
3	Control Data G-15	Control Data Corporation 5030 Arbor Vitae Street Los Angeles 45, Calif.	863 activities (CPM) 431 activities (PERT)	basic CPM or PERT without calendar dates or alphabetic descriptions; one output report
4	Control Data G-20	same as above	150 to 2000+ activi- ties, depending on core and tape capacity	"PERT I" is a basic PERT program, with limited output capability
5	Control Data G-20	same as above	same as above	"PERT II" is a basic PERT program, with a variety of output sorts and formats
6	Control Data 1604	Mr. Ronald L. Benton Computer Division Control Data Corporation 8100 34th Avenue South Minneapolis 20, Minnesota	3000 activities	basic PERT
7	GE-225	Mr. Earnest Nussbaum Washington Information Process- ing Center General Electric Company 7800 Wisconsin Avenue Bethesda 14, Maryland	2100 activities 1000 events	basic CPM with random numbering of events; calendar dates; cost optimization; 15 optional sorts; tape file maintenance; called "Critical Path Method Program" and "Project Monitor and Control Method (PROMOCOM)"
8	Honeywell 400 or 1400	Honeywell Electronic Data Proc- essing 60 Walnut Street Wellesley Hills 81, Mass.	3000 activities 2000 events	basic PERT
9	Honeywell 800 or 1800	same as above	unlimited	basic PERT; slack computed from inter- mediate scheduled dates
10	IBM 650	Mr. F. Backer, Jr. IBM Applied Science 2911 Cedar Springs Road Dallas 19, Texas	999 events	basic CPM with cost optimization; called "LESS" program; 650 Program Library No. 103.005

<sup>1</sup>Joseph J. Moder and Cecil R. Phillips, Project Management with CPM and PERT (New York: Reinhold Publishing Corporation, 1964), pp. 255-260.



<i>No.</i>	<i>Computer Equipment</i>	<i>Information on Program Contact for Further</i>	<i>Capacity</i>	<i>Category and Comments</i>
11	IBM 650 or 1620	Mr. H. N. Perk Systems Analysis Dept. The Dow Chemical Co. Freeport, Texas	capacity is problem dependent	"manscheduling" addition to LESS program above allows limitations on availabilities of up to 10 skill categories; 650 Program Library No. 10.3.006., 1620 Program Library No. 10.3.013
12	IBM 704 or 7090	Mr. F. B. Quackenboss, Head Operations Research Dept. General Motors Research Laboratories 12 Mile and Mound Roads Warren, Michigan	—	basic CPM with cost optimization; 704 Program Library No. 1188GMCP
13	IBM 1401	TYPHON Weapon System Analysis Center Johns Hopkins Univ./APL 8621 Georgia Avenue Silver Spring, Maryland	unlimited	several programs for validation and editing of input data prior to processing on APL/JHU version of NASA PERT "B" program for IBM 7090
14	IBM 1401	Mr. Lou Granato IBM Corporation 631 Cooper Street Camden 2, New Jersey	985 to 2125 events, depending on core	basic CPM
15	IBM 1410	Mr. Keith Woodcox IBM Corporation 1371 Peachtree St., N.E. Atlanta 9, Georgia	1000 events, or 3397 events and activities	basic CPM but with PERT features of probability and scheduled dates
16	IBM 1620 (card)	Mr. John C. Patton IBM Corporation 1220 19th Street, N.W. Washington 6, D. C.	999 events	PERT and CPM with cost optimization 1620 Program Library No. 10.3.006
17	IBM 1620 (tape)	Mr. Bob N. Manning Goodyear Aircraft Corp. Arizona Division Litchfield Park, Arizona	—	basic PERT, 1620 Program Library No. 10.3.009
18	IBM 1620 (card or tape)	Mr. Ray N. Sauer IBM Corporation 2601 South Main Street Houston, Texas	Total of 1614 events and activities for 20K equipment, 3614 for 40K, and 5614 for 50K	basic CPM with random event numbering and random activity ordering; 1620 Program Library Nos. 10.3.011 (card) and 10.3.012 (tape)
19	IBM 7070 and 1401	Miss Shirley Inman Computer Techniques Dept. Collins Radio Company	—	basic PERT, with combination activity and event orientation



No.	Computer Equipment	Contact for Further Information on Program	Capacity	Category and Comments
20	IBM 7090	Commander U. S. Naval Weapons Lab. Dahlgren, Virginia Attn: Code KPO	13,500 events	basic PERT; file establishment and file updating by exception on magnetic tape included in the program; activity and event reports generated
21	IBM 7090	Mr. Walter W. Shirley 645 South Mariposa Ave. Los Angeles 5, Calif.	1750 to 8000 activities, depending on equipment	basic CPM with "manscheduling" of up to 99 crafts per network; six crafts per activity; 7090 Program Library No. 1320R08000
22	IBM 7090	PERT Control Board Hq. AFSC (SCCS) Andrews AFB Washington 25, D. C.	12,000 activities in detailed network	basic PERT with network condensation and integration; activity and/or event orientation; tape file maintenance; optional treatments of scheduled dates; graphical output for selected milestones; called "PERT, III"
23	IBM 7090	Mr. R. W. Haine, Mgr. Management Information Sys. Corporate Industrial Dynamics Hughes Aircraft Company International Airport Station P. O. Box 90515 Los Angeles 9, Calif.	5120 activities in detailed network	basic PERT with cost control; network condensation and integration; tape file maintenance; graphical output for selected milestones; several levels of cost summarization; program is proprietary
24	IBM 7090/94	C-E-I-R, INC. Marketing Information Services 1200 Jefferson Davis Hwy. Arlington, Virginia 22202	approximately 700 activities total for 3 projects and 20 resources (it is a function of the mix of these parameters)	basic CPM with resource allocation and multiproject scheduling; proprietary system, called "RAMPS"; allocates limited resources to competing activities on the basis of several management-controlled features including float factors, project priorities and delay penalties; handles varying resource availability, shift operations, several accomplishment rates, and work efficiencies; producing a work Schedule, resource summary and cost report
25	IBM 7090/94 and Gen. Dynamics Electronics SC4020	Mr. K. Leon Montgomery Secretary UAIDE General Dynamics/Astronautics P. O. Box 166 Mail Zone 591-50 San Diego 12, Calif.	up to 1000 activities or more, depending on tape capacity	called "NAP," system accepts PERT network charts via the cathode ray tube computer (SC4020); series of pictures must be joined together to produce complete network
26	IBM 7090/94	Local IBM Branch Office	75,000 activities	basic PERT with cost optimization and control; up to seven levels of summarization; also included are computer generated networks and time, manpower, and cost summaries; called "PERT Cost," Lib. No. 7090-CP-01X
27	IBM 7090/94	NASA Headquarters Management Information Systems Division, Office of Administration Washington 25, D. C.	7168 activities	has most features of PERT class except probability; accepts only single time estimate; summarizes by milestones; master file stored on cards or tape with updating to file by 1401; called "NASA PERT B"
28	IBM 7090 (optional use of IBM 1401)	Douglas Aircraft Co., Inc. 3000 Ocean Park Boulevard Santa Monica, California Attn: Aerospace Computing, A2-260	4000 activities per detail network—up to 125 detail networks and/or 50,000 activities	basic features of PERT except probability; network summarization and skeletonization capabilities; graphical output and optional generation of printed gummed labels for use in network construction; error check capability includes optional use of 1401 pre-processor edit program; called "Douglas PERT MARK IV System" Proprietary
29	NCR-304 or NCR-315	Dr. E. W. Bold Scientific Programming DPS&S National Cash Register Dayton 9, Ohio	5000 activities	basic PERT



<i>No.</i>	<i>Computer Equipment</i>	<i>Contact for Further Information on Program</i>	<i>Capacity</i>	<i>Category and Comments</i>
30	Philco 2000	Mr. R. E. Larson Philco Corporation Western Computing Center 3875 Fabian Way Palo Alto, Calif.	750 activities	basic PERT; called "WCC PERT"
31	RECOMP II	Autonetics, Computers & Data Systems 3370 East Anaheim Road Anaheim, California	703 activities	basic CPM; accepts three time estimates and multiple initial nodes; several optional outputs; Program No. 58, "SCHEDULE CRITICAL PATH"
32	RCA 301	Management Science Radio Corporation of America Electronic Data Processing Division Cherry Hill, Camden 8, N. J.	16,000 activities 9000 events	basic PERT with several sort options and report codes; time unit arbitrary when unit is available; intermediate schedule dates are optional for slack computation
33	RCA 301	same as above.	24,000 activities	basic resource allocation for up to 50 different resources, plus aggregate cost analysis; considers availability of resources by priority and costs; graphical outputs
34	RCA 501	same as above.	2000 activities 1000 events	basic PERT; accepts punched cards, paper tape, or magnetic tape
35	RCA 501 or RCA 301	same as above.	—	called "APEX," a system that integrates accounting entries with project control reporting; provides much of the information for network cost control
36	UNIVAC 1107	V. Erskine UNIVAC Division Sperry Rand Corp. Sperry Rand Bldg. New York 19, New York	12,000 activities 12,000 events	basic PERT with cost control called "UNIVAC 1107 PERT Cost System"; Satisfies reporting requirements of DOD/NASA; contains network condensation and integration feature









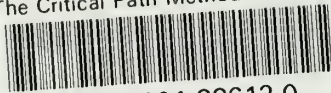




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